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# KUESTION



**CONTROL SYSTEMS**

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## Manual for Kuestion

### Why Kuestion?

It's very overwhelming for a student to even think about finishing 100-200 questions per chapter when the clock is ticking at the last moment. This is the reason why Kuestion serves the purpose of being the bare minimum set of questions to be solved from each chapter during revision.

### What is Kuestion?

A set of 40 questions or less for each chapter covering almost every type which has been previously asked in GATE. Along with the Solved examples to refer from, a student can try similar unsolved questions to improve his/her problem solving skills.

### When do I start using Kuestion?

It is recommended to use Kuestion as soon as you feel confident in any particular chapter. Although it will really help a student if he/she will start making use of Kuestion in the last 2 months before GATE Exam (November end onwards).

### How do I use Kuestion?

Kuestion should be used as a tool to improve your speed and accuracy chapter wise. It should be treated as a supplement to our K-Notes and should be attempted once you are comfortable with the understanding and basic problem solving ability of the chapter. You should refer K-Notes Theory before solving any "Type" problems from Kuestion.



## Type 1: Transfer Function

For Concept, refer to Control Systems K-Notes, Basics of Control Systems

Pre-requisites: Laplace Transform (Signals and Systems)

### Sample Problem-1:

A linear time invariant system initially at rest, when subjected to a unit step input, gives a response  $y(t) = te^{-t}$ ,  $t > 0$ . The transfer function of the system is

- (A)  $\frac{1}{(s+1)^2}$  (B)  $\frac{1}{s(s+1)^2}$  (C)  $\frac{s}{(s+1)^2}$  (D)  $\frac{1}{s(s+1)}$

**Solution:** (C) is correct option.

Step input  $= u(t)$

Output  $y(t) = te^{-t}$

$$\therefore TF = \frac{L[\text{output}]}{L[\text{input}]} = \frac{L[te^{-t}u(t)]}{L[u(t)]}$$

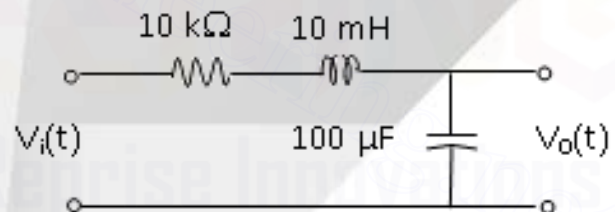
$$\therefore TF = \frac{\frac{1}{(s+1)^2}}{\frac{1}{s}} = \frac{s}{(s+1)^2}$$

### Unsolved Problems:

**Q.1** For the circuit shown in figure, the initial conditions are zero. Its transfer function

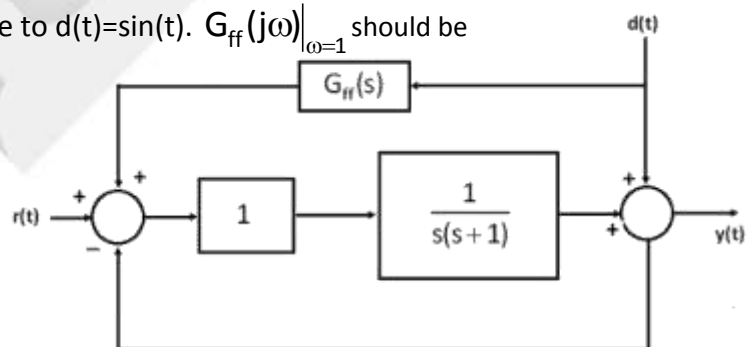
$$H(s) = \frac{V_o(s)}{V_i(s)} \text{ is?}$$

- (A)  $\frac{1}{s^2 + 10^6 s + 10^6}$  (B)  $\frac{10^6}{s^2 + 10^3 s + 10^6}$   
(C)  $\frac{10^3}{s^2 + 10^3 s + 10^6}$  (D)  $\frac{10^6}{s^2 + 10^6 s + 10^6}$



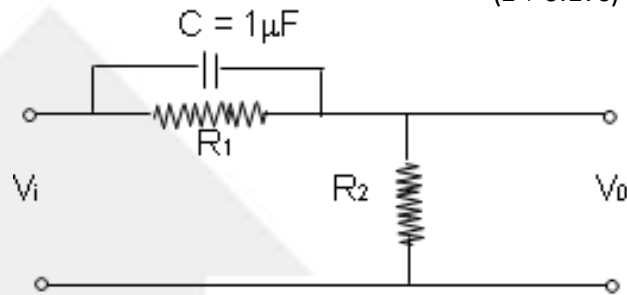
**Q.2** A disturbance input  $d(t)$  is injected into the unity feedback control loop shown in the figure. Take the reference input  $r(t)$  to be unit step. If the disturbance is measureable, its effect on the output can be minimized significantly using a feed forward controller  $G_{ff}(s)$ . To eliminate the component of the output due to  $d(t) = \sin(t)$ ,  $G_{ff}(j\omega)|_{\omega=1}$  should be

- (A)  $\frac{1}{\sqrt{2}} \angle -\frac{3\pi}{4}$  (B)  $\frac{1}{\sqrt{2}} \angle \frac{\pi}{4}$   
(C)  $\sqrt{2} \angle \pi$  (D)  $\sqrt{2} \angle -\frac{\pi}{4}$



**Q.3** The transfer function to the given electrical network, shown in figure, is  $\frac{K(1+0.3s)}{(1+0.17s)}$  The values of  $R_1$  and  $R_2$  are respectively.

- (A)  $300k\Omega$  and  $300k\Omega$
- (B)  $300k\Omega$  and  $400k\Omega$
- (C)  $400k\Omega$  and  $300k\Omega$
- (D)  $400k\Omega$  and  $400k\Omega$



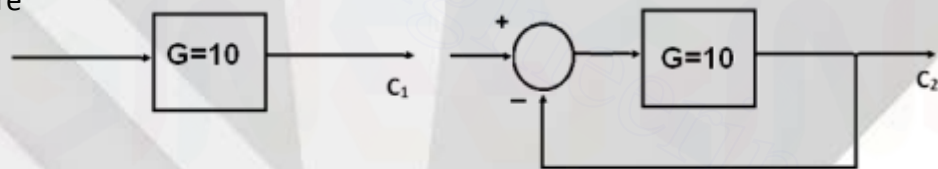
**Q.4** For the given system below, the feedback does not reduce the closed loop sensitivity due to variation of which one of the following?

- (A)  $K$
- (B)  $A$
- (C)  $\alpha$
- (D)  $\beta$



**Q.5** For the systems shown in figure below if the  $G$  changes by 10% , the % changes in  $C_1$  and  $C_2$  respectively are

- (A) 10%, 1%
- (B) 10%, 2%
- (C) 5%, 3%
- (D) 5%, 10%



**Q.6** The response  $y(t)$  of a linear system to an excitation  $x(t)=e^{-3t}u(t)$  is  $y(t)=(2t+1)e^{-2t}u(t)$ . Poles and zeroes will be at?

- (A) -1,-1 and -2,-2
- (B) -2,-2 and -3,-4
- (C) -3,-3 and -4,-5
- (D) None of these

## Type 2: Signal Flow Graph and Block Diagram Algebra

For Concept, refer to Control Systems K-Notes, Signal Flow Graphs

### Common Mistakes:

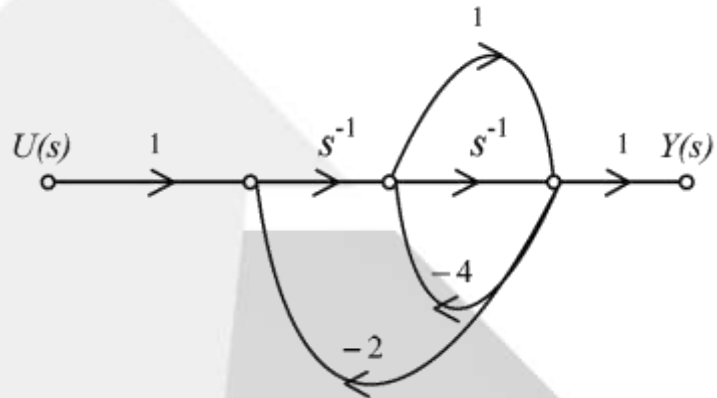
If two touching loops have been counted individually then they must not be counted as part of a single loop again.

The input to a signal flow graph cannot be connected to any loops it should have only one outgoing branch. If no such input is given we have to assume a hypothetical input. (Refer Q.5)

## Sample Problem 2:

The signal flow graph for a system is given below. The transfer function  $\frac{Y(s)}{U(s)}$  for this system is

- (A)  $\frac{s+1}{5s^2+6s+2}$   
 (B)  $\frac{s+1}{s^2+6s+2}$   
 (C)  $\frac{s+1}{s^2+4s+2}$   
 (D)  $\frac{1}{5s^2+6s+2}$



**Solution:** (A) is correct option

Forward paths

$$P_1 = U - X_1 - X_2 - X_3 - Y = S^{-2}$$

$$P_2 = U - X_1 - X_2 - X_3 - Y = S^{-1}$$

Loops:

$$L_1 = X_1 - X_2 - X_3 - X_1 = -2S^{-2}$$

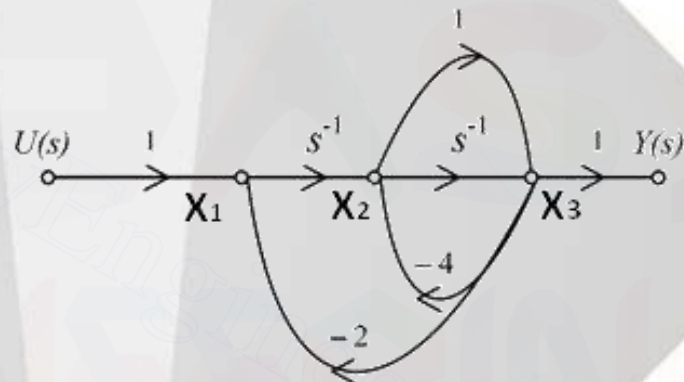
$$L_2 = X_1 - X_2 - X_3 - X_1 = -2S^{-1}$$

$$L_3 = X_2 - X_3 - X_2 = -4S^{-2}$$

$$L_4 = X_2 - X_3 - X_2 = -4$$

$$\Delta_1 = 1, \Delta_2 = 1$$

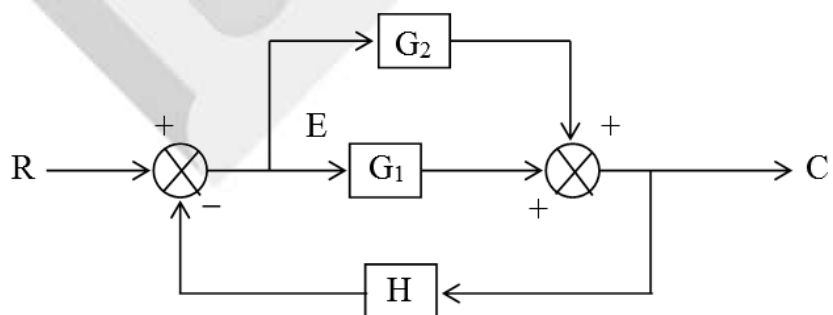
$$\begin{aligned} \text{T.F.} &= \frac{s^{-2} + s^{-1}}{1 - (-2s^{-2} - 2s^{-1} - 4s^{-1} - 4)} \\ &= \frac{s+1}{5s^2+6s+2} \end{aligned}$$



## Unsolved Problems:

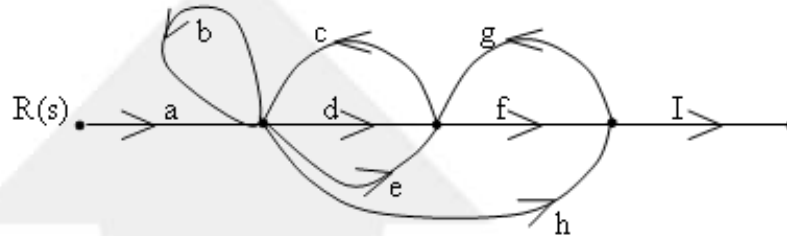
**Q.1** The transfer function relating E and R for the block diagram given below is

- (A)  $\frac{1}{1+(G_1+G_2)H}$   
 (B)  $\frac{1}{1+G_1G_2H}$   
 (C)  $\frac{G_1G_2}{1+(G_1+G_2)}$   
 (D)  $\frac{G_1G_2}{1+G_1G_2H}$



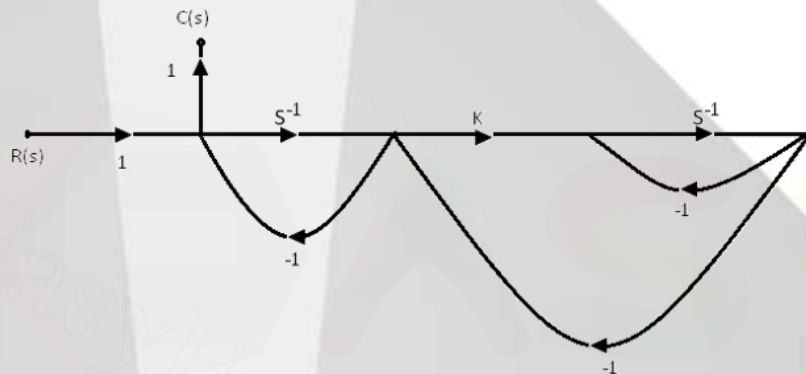
**Q.2** The number of forward paths and the number of non-touching loop pairs for the signal flow graph given in the figure are, respectively,

- (A) 1, 3
- (B) 3, 2
- (C) 3, 1
- (D) 2, 4



**Q.3** Fig shows the signal flow graph of the system. Find the transfer function  $\frac{C(s)}{R(s)}$  = ?

- (A)  $\frac{K+1}{s^2 + (K+1)s + 1}$
- (B)  $\frac{s(K+1)}{s^2 + (K+2)s + 1}$
- (C)  $\frac{s}{s^2 + (K+1)s + 1}$
- (D)  $\frac{s(K+1)}{s^2 + 2s + 1}$

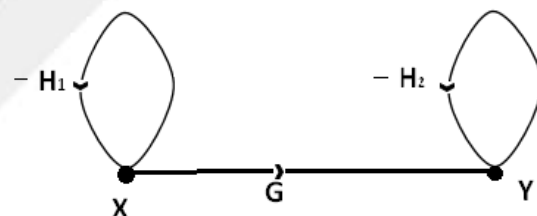


**Q.4** Three blocks  $G_1$ ,  $G_2$  &  $G_3$  are connected in some fashion such that overall transfer function is  $\frac{G_1 + G_3(1 + G_1G_2)}{1 + G_1G_2}$ . The blocks are connected in the following manner

- (A)  $G_1$ ,  $G_2$  with negative feedback and combination in parallel with  $G_3$
- (B)  $G_1$ ,  $G_3$  with negative feedback and  $G_2$  in parallel
- (C)  $G_1$ ,  $G_2$  in cascade & combination in parallel with  $G_3$
- (D)  $G_1$ ,  $G_3$  in cascade & combination in parallel with  $G_2$

**Q.5** find Y/X

- (A)  $\frac{G}{1+H_1}$
- (B)  $\frac{GH_2}{1+H_1}$
- (C)  $\frac{G}{1+H_2}$
- (D)  $\frac{GH_1}{1+H_2}$





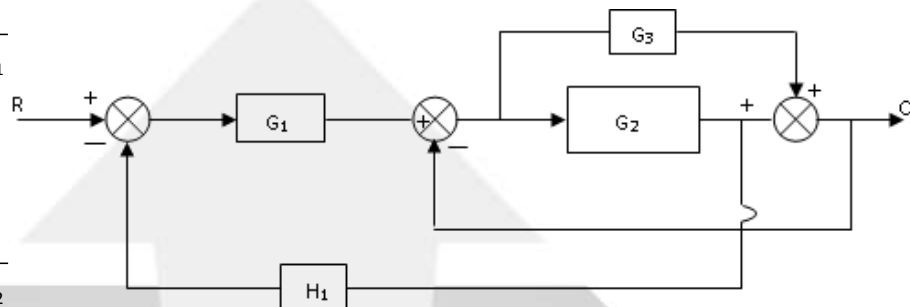
**Q.6** The overall transfer function relating C and R for the system whose block diagram as shown in figure is

(A)  $\frac{G_1 G_2 + G_1 G_3}{1 + G_2 + G_3 + G_1 G_2 H_1}$

(B)  $\frac{G_1 G_3 + G_2 H_1}{1 + G_2 G_3 H_1 + G_1 G_3}$

(C)  $\frac{G_2 G_3 + G_1 H_1}{1 + G_2 G_1 H_1 + G_3 + G_2}$

(D)  $\frac{G_1 G_3 + G_3 H_1}{1 + G_2 G_1 H_1 + G_3 + G_2}$



## Type-3: Routh Stability Criterion

For Concept, refer to Control Systems K-Notes Control System Stability

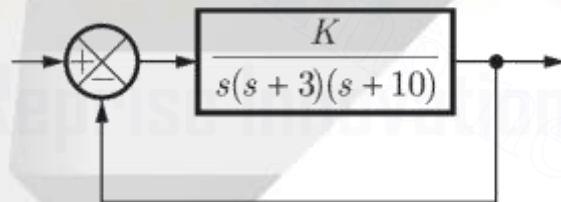
### Common Mistake:

A row of zeroes does not guarantee the existence of poles on  $j\omega$  axis, we must first calculate the roots of auxiliary polynomial and then decide.

### Sample Problem 3:

Figure shows a feedback system where  $K > 0$ , the range of  $K$  for which the system is stable will be given by

- (A)  $0 < K < 30$   
 (B)  $0 < K < 39$   
 (C)  $0 < K < 390$   
 (D)  $K > 390$



**Solution:** (C) is correct option .

Characteristic equation for the system

$$1 + \frac{K}{s(s+3)(s+10)} = 0$$

$$s(s+3)(s+10) + K = 0$$

$$s^3 + 13s^2 + 30s + K = 0$$

Applying Routh's stability criteria.

For stability there should be no sign change in first column

So,  $390 - K > 0$  &  $K < 390$

$\therefore K > 0$  ,  $\therefore 0 < K < 390$

$S^3$	1	30
$S^2$	13	K
$S^1$	$\frac{(13 \times 30) - K}{13}$	
$S^0$	K	

### Unsolved Problems:

**Q.1** The open loop transfer function of a system is  $G(s)H(s) = \frac{K(s+4)}{s(s^2+2s+2)}$ , the root locus will intersect the imaginary axis at

- (A)  $\pm j2$  (B)  $\pm j\sqrt{2}$  (C)  $\pm j\sqrt{10}$  (D)  $\pm j\sqrt{102}$

**Q.2** The Routh – Hurwitz array is given for a third order characteristic equation as follows

$S^3$	a	b	0
$S^2$	c	d	0
$S^1$	$\frac{10-k}{2}$	0	
$S^0$	k		

The values of a, b, c, d are respectively, for which the system should be marginally stable

- (A) 1 5 2 10 (B) 2 10 1 5  
(C) 10 1 2 5 (D) 1 2 5 10

**Q.3** The open loop transfer function of a control system is given by  $\frac{K(s+10)}{s(s+2)(s+\alpha)}$ . The smallest possible value of “ $\alpha$ ” for which this system is stable in the closed-loop for all positive values of k is

- (A) 0 (B) 8 (C) 10 (D) 12

**Q.4** The no of right hand, left hand, and  $j\omega$  axis poles are respectively  $T(s) = \frac{s^2+7s+10}{s^6+2s^4-s^2-2}$

- (A) 3,1,2 (B) 2,1,2 (C) 4,0,2 (D) 1,1,4

**Q.5** Consider the characteristic equation  $D(s) = s^5 + S^4 + 3s^3 + 3s^2 + 6s + 4$ . Then system is stable or unstable and if unstable, then how many poles lie in the right half of s-plane.

- (A) Unstable, 3 poles lie in the right half of s plane  
(B) Stable, 0 poles lie in the right half of s plane  
(C) Unstable, 2 poles lie in the right half of s plane  
(D) None of these

## Type 4: Time Response Analysis

For Concept, refer to Control Systems K-Notes, Time Response Analysis

### Sample Problem 4:

For the system shown in figure, with a damping ratio  $\xi$  of .7 and an un-damped natural frequency  $\omega_n$  of 4 rad/sec, the values of 'K' and 'a' are

- (A)  $K=4, a=.35$  (B)  $K=8, a=.455$   
 (C)  $K=16, a=.225$  (D)  $K=64, a=.9$



**Solution:** (C) is correct option

Characteristic equation

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(1+as)}{(s+2)} = 0$$

$$s^2 + 2s + Kas + K = 0$$

$$s^2 + s(Ka + 2) + K = 0$$

Compare with standard equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\therefore \omega_n^2 = K = 4^2 = 16$$

$$K = 16$$

$$Ka + 2 = 2\xi\omega_n$$

$$= 2(.7)(4)$$

$$a = \frac{2(.7)(4) - 2}{16} = .225$$

$$a = .225$$

### Unsolved Problems:

**Q.1** A controller of the form  $G_c(s) = \frac{K(s+a)}{(s+b)}$  is designed for a plant with transfer function

$G(s) = \frac{1}{s(s+3)}$  such that the un-damped natural frequency and the damping ratio of the closed

loop 2nd order system are 2 rad/sec and 0.5 respectively. When the steady state error to a unit step input is zero, then the controller parameters are

- (A)  $k = 4, a = 3, b = 2$  (B)  $k = 1, a = 3, b = 2$   
 (C)  $k = 4, a = 2, b = 3$  (D)  $k = 1, a = 2, b = 3$

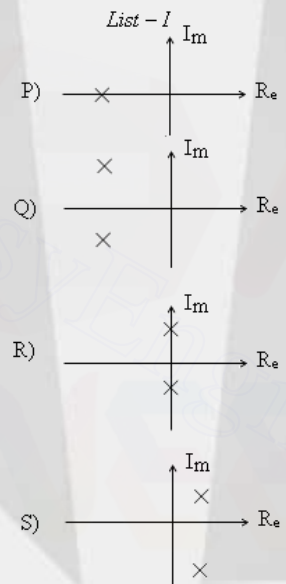
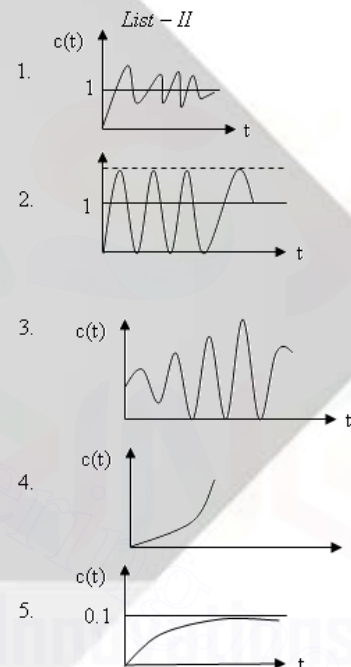
**Q.2** A feedback system has un-damped frequency of 10 rad/sec and the damping ratio is 0.4 the transfer function is?

- (A)  $\frac{100}{(s^2 + 8s + 100)}$  (B)  $\frac{10}{(s^2 + 8s + 10)}$  (C)  $\frac{100}{(s^2 + 4s + 100)}$  (D)  $\frac{10}{(s^2 + 8s + 100)}$

**Q.3** A system is required to have a peak overshoot of 4.32 % and a natural frequency of 5 rad/sec. The required location of the dominant pole is:

- (A)  $-3.535 + j 5.535$  (B)  $-2.535 + j 2.525$  (C)  $-2.535 + j 3.535$  (D)  $-3.535 + j 3.535$

**Q.4** Match List – I (Location of roots) with List – II (Step response) and select the correct answer using the codes given below the lists:

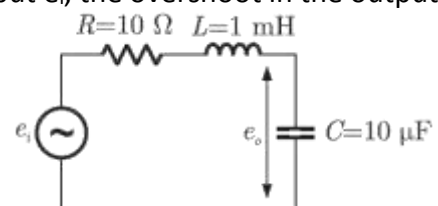
	P	Q	R	S	List - I 	List - II 
(A)	4	1	2	3		
(B)	5	1	2	3		
(C)	4	2	1	3		
(D)	5	2	3	4		

**Q.5** The open loop transfer function of a system is  $G(s) = \frac{100}{(s+1)(s+100)}$  for a unit step input to the system, the approximate settling time for 2% criterion is

- (A) 100 sec (B) 4 sec (C) 1 sec (D) none

**Q.6** Consider the R-L-C circuit shown in figure For a step-input  $e_i$ , the overshoot in the output  $e_o$  will be

- (A) 0 (B) 5 %  
(C) 16 % (D) 48 %





## Type 5: Root-Locus Technique

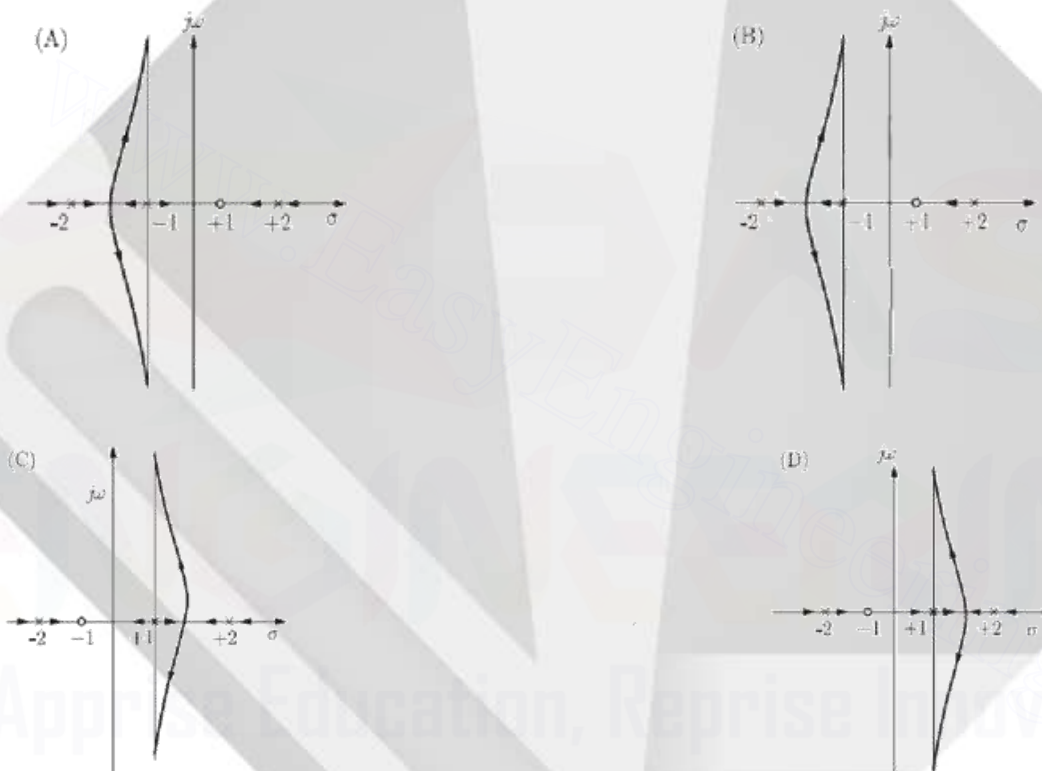
For Concept, refer to Control Systems K-Notes, Root Locus Technique

### Common Mistake:

For positive feedback the properties of root-locus are reversed. Refer GATE-2014 EE-01 Solutions Ques-18

### Sample Problem 5:

A closed-loop system has the characteristic function  $(s^2 - 4)(s + 1) + K(s - 1) = 0$ . Its root locus plot against  $K$  is



**Solution:** (C) is correct option

Closed loop transfer function of the given system is,

$$T(s) = \frac{s^2 + 4}{(s + 1)(s + 4)}$$

$$T(j\omega) = \frac{j\omega^2 + 4}{(j\omega + 1)(j\omega + 4)}$$

If system output is zero

$$|T(j\omega)| = \frac{|4 - \omega^2|}{|(j\omega + 1)(j\omega + 4)|} = 0$$

$$4 - \omega^2 = 0$$

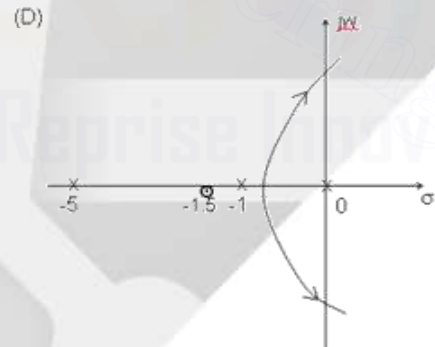
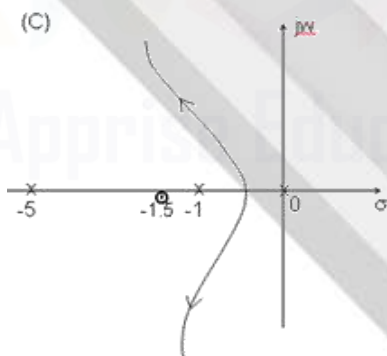
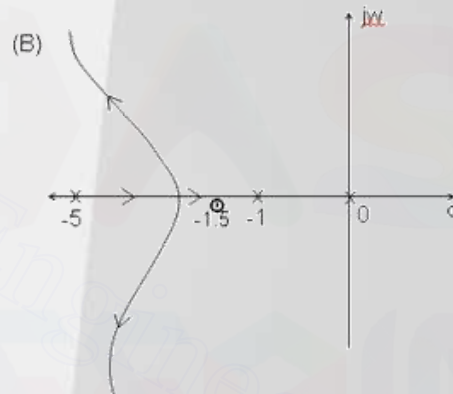
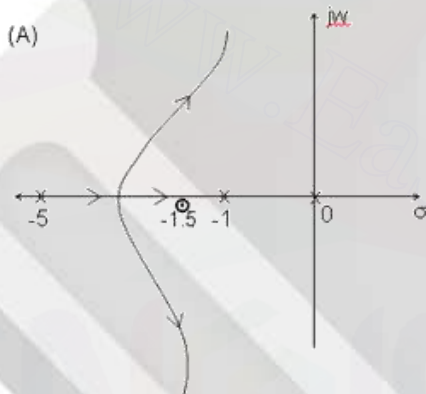
$$\omega^2 = 4$$

$$\omega = 2 \text{ rad/sec}$$

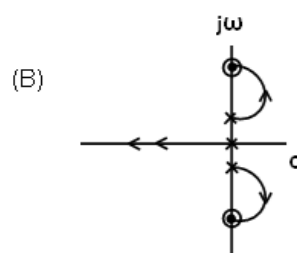
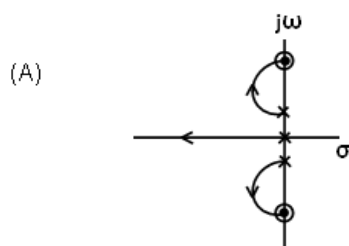
**Unsolved Problems:**

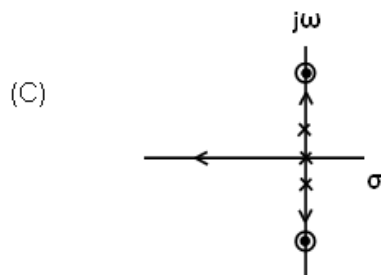
**Q.1** Which one of the following is the root locus of a open loop transfer function of

$$G(s)H(s) = \frac{K(s + 1.5)}{s(s + 1)(s + 5)}$$



**Q.2.** OLTF of an unity feedback system is  $\frac{K(s^2 + 10)}{s(s^2 + 4)}$ . The RLD is



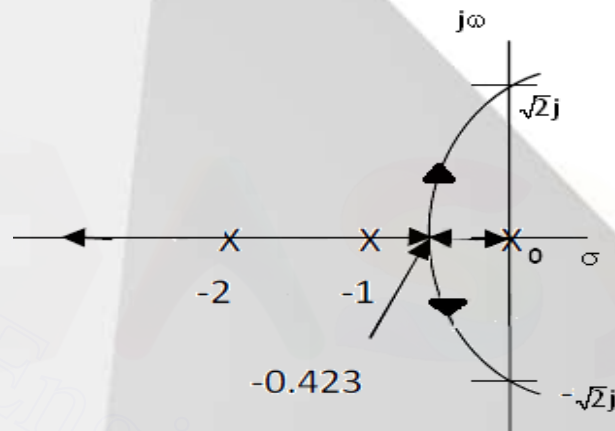


(D) None of the above

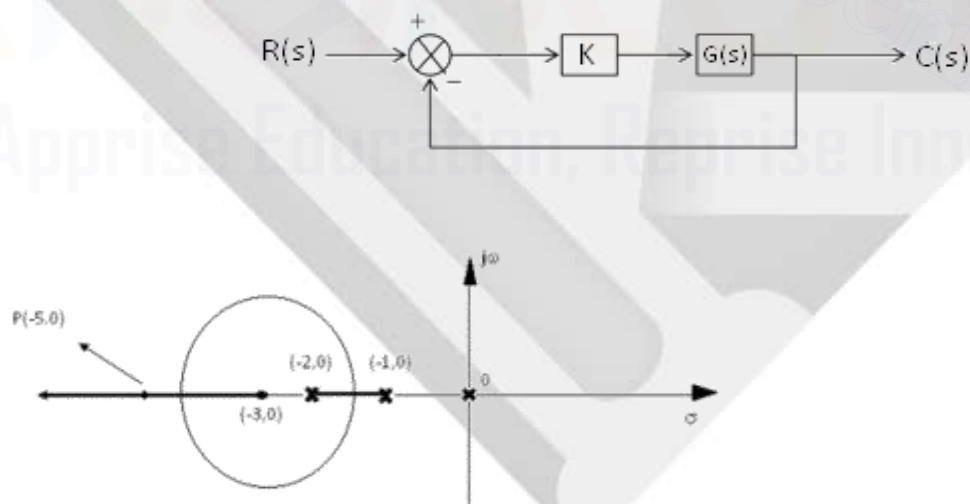
**Q.3** The root locus plot for a system, with transfer function  $\frac{2}{s(s+1)(s+2)}$ , is shown in Figure.

A unity feedback proportional control system is built using this system. The maximum possible controller gain, for which the unity feedback system is stable, is approximately

- (A) 6.0
- (B) 3.0
- (C) 0.4
- (D) 0.2



**Q.4** A unity feedback system is shown in Fig. below. The root-locus of its characteristic equation is shown in Fig. below

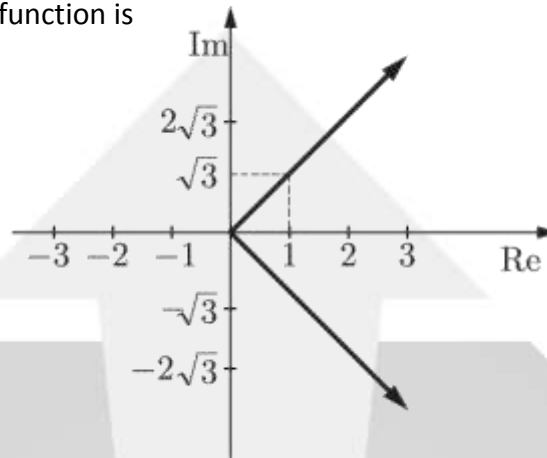


If P (-5, 0) is a point on the root-locus, the variable parameter K of the system at P is?

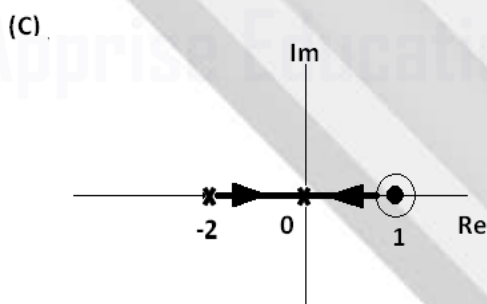
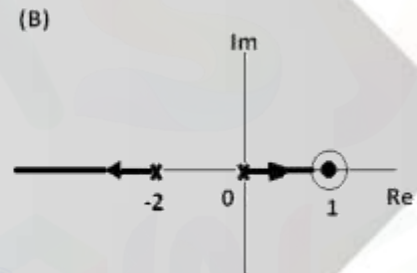
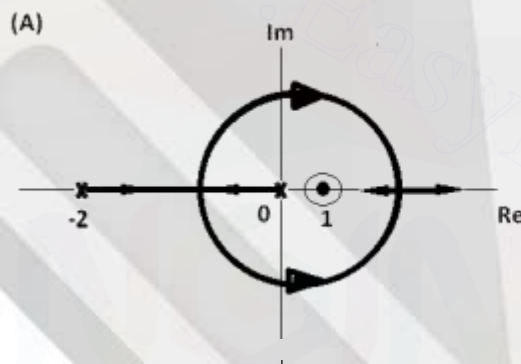
- (A) 2
- (B) 4
- (C) 6
- (D) 8

**Q.5** Figure shows the root locus plot (location of poles not given) of a third order system whose open loop transfer function is

- (A)  $\frac{K}{s^3}$   
 (B)  $\frac{K}{s^2(s+1)}$   
 (C)  $\frac{K}{s(s^2+1)}$   
 (D)  $\frac{K}{s(s^2-1)}$



**Q.6** RLD of the system with loop TF  $\frac{K(1-s)}{s(s+2)}$ , when K is varied from 0 to  $\infty$  is



(D) None of these



## Type 6: Polar Plot

For Concept, refer to Control Systems K-Notes, Frequency Response Analysis

### Common Mistake:

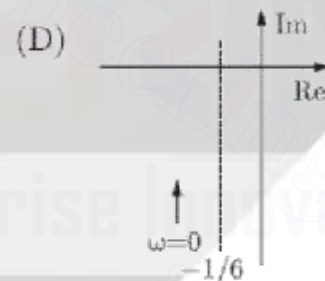
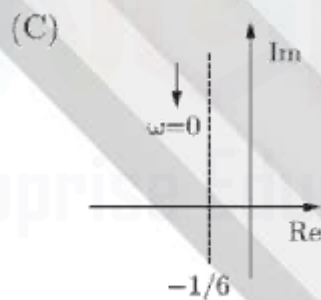
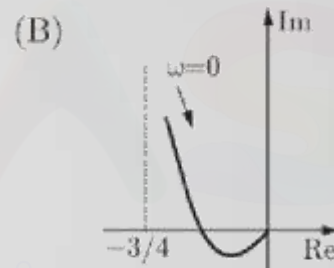
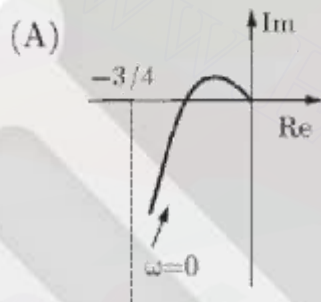
Remember the direct polar plot but also remember how to calculate the intersection with real axis as you may be given two options with same shape of polar plot but different intersection point.

### Sample Problem 6:

The frequency response of

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

plotted in the complex  $G(j\omega)$  plane (for  $0 < \omega < 3$ ) is



**Solution:** (A) is correct option

Given  $G(s) = \frac{1}{s(s+1)(s+2)}$

$$G(s) = \frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

In Nyquist plot

For  $\omega=0$   $|G(j\omega)| = \infty$

$$\angle G(j\omega) = -90^\circ$$

Intersection at real axis

$$\begin{aligned} G(s) &= \frac{1}{j\omega(j\omega+1)(j\omega+2)} \\ &= \frac{1}{j\omega(-\omega^2+3j\omega+2)} \\ &= \frac{-3\omega^2}{9\omega^4+\omega^2(2-\omega^2)^2} - \frac{j\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2} \end{aligned}$$

At real axis

$$\text{Im}[G(j\omega)] = 0$$

$$\text{So, } \frac{\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2} = 0$$

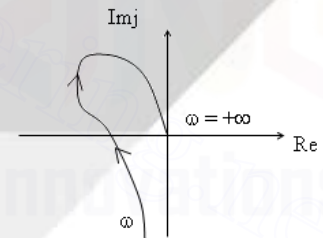
$$(2-\omega^2) = 0 \Rightarrow \omega = \sqrt{2} \text{ rad/sec}$$

At  $\omega=\sqrt{2}$  rad/sec, magnitude response is

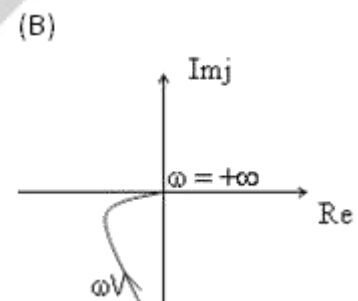
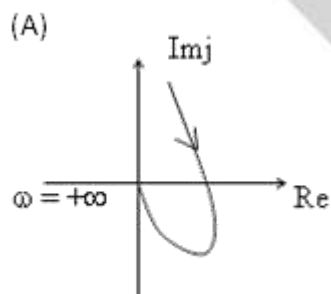
$$|G(j\omega)|_{\text{at } \omega=\sqrt{2}} = \frac{1}{\sqrt{2}\sqrt{2+1}\sqrt{2+4}} = \frac{1}{6} < \frac{3}{4}$$

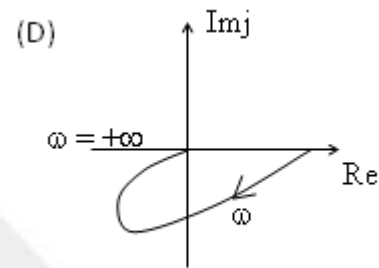
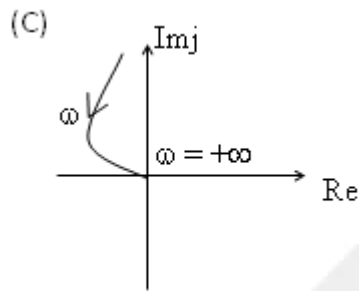
### Unsolved Problems:

**Q.1** The polar plot for a transfer function is shown below:



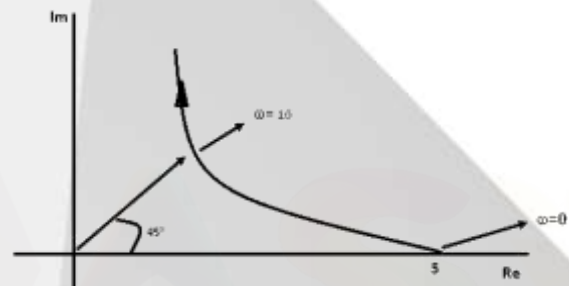
An addition of a zero will change the plot as



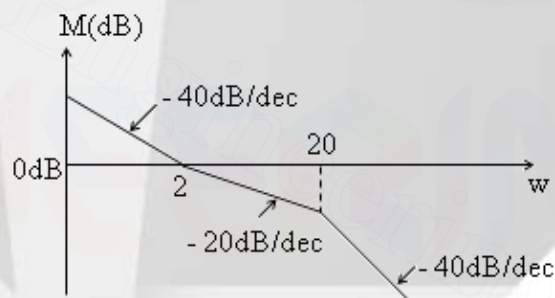


**Q.2** Fig shows the polar plot of a system. The transfer function of the system is

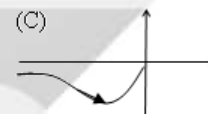
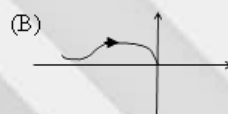
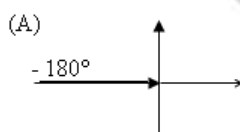
- (A)  $5(1 + 0.1s)$
- (B)  $(1 + 0.5s)$
- (C)  $5(1 + 10s)$
- (D)  $5(1 + s)$



**Q.3** The magnitude plot for minimum phase system is shown in figure



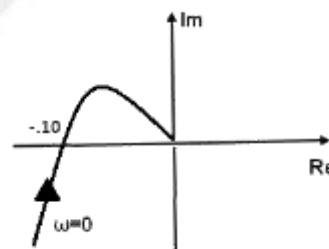
The polar plot of the above system is



(D) None

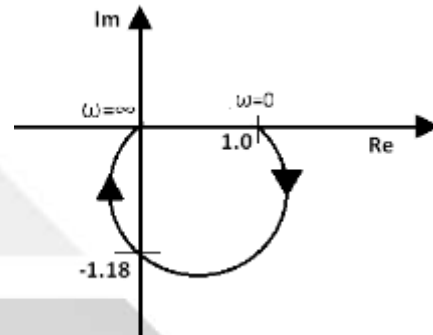
**Q.4** The polar plot of  $G(s)H(s)$  for  $K=10$  is given below . The range of 'K' for stability is?

- (A)  $0 < K < 10$
- (B)  $0 < K < 10^3$
- (C)  $0 < K < 10^2$
- (D)  $0 < K < 1$



**Q.5** The polar plot of frequency response of a linear damped second order system is shown in the figure given. What is the transfer function of this system?

- (A)  $\frac{8}{s^2 + 10s + 1}$   
 (B)  $\frac{8}{s^2 + 8.48s + 10}$   
 (C)  $\frac{100}{s^2 + 8.48s + 100}$   
 (D)  $\frac{100}{s^2 + 10s + 8.48}$



## Type 7: Bode Plot

For Concept refer to Control System K-Notes, Bode Plots

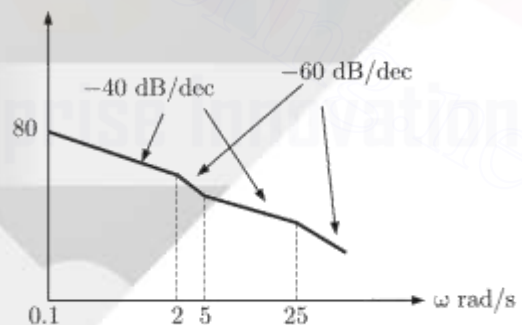
### Common Mistake:

Remember the x-axis is always in terms of  $\log \omega$  and not in terms of  $\omega$ , this is a common mistake while calculating slope.

### Sample Problem 7:

The asymptotic approximation of the log-magnitude v/s frequency plot of a system containing only real poles and zeros is shown. Its transfer function is

- (A)  $\frac{10(s+5)}{s(s+25)(s+2)}$   
 (B)  $\frac{1000(s+5)}{s^2(s+25)(s+2)}$   
 (C)  $\frac{100(s+5)}{s(s+25)(s+2)}$   
 (D)  $\frac{80(s+5)}{s^2(s+25)(s+2)}$



**Solution:** (B) is correct option

Since initial slope of the bode plot is  $-40$  dB/decade, so no. of poles at origin is 2.

Transfer function can be written in following steps:

- Slope changes from  $-40$  dB/dec. to  $-60$  dB/dec. at  $\omega_1 = 2$  rad/sec., so at  $\omega_1$  there is a pole in the transfer function.
- Slope changes from  $-60$  dB/dec to  $-40$  dB/dec at  $\omega_2 = 5$  rad/sec., so at this frequency there is a zero lying in the system function.



- The slope changes from  $-40$  dB/dec to  $-60$  dB/dec at  $\omega_3 = 25$  rad/sec, so there is a pole in the system at this frequency.

Transfer Function

$$T(s) = \frac{K(s+5)}{s^2(s+25)(s+2)}$$

Constant term can be obtained as:

$$T(j\omega)|_{\omega=0.1} = 80$$

$$\text{So, } 80 = 20 \log \left[ \frac{K(5)}{(.1)^2 \times 50} \right]$$

$$K = 1000$$

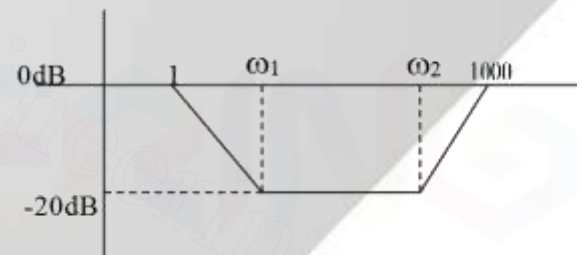
therefore the Transfer Function is

$$T(s) = \frac{1000(s+5)}{s^2(s+25)(s+2)}$$

### Unsolved Problems:

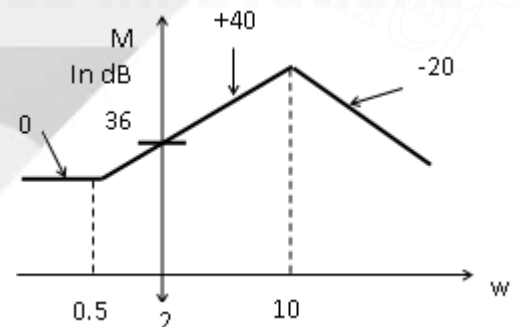
**Q.1** The Bode plot of a system is given its transfer function is

- (A)  $\frac{(1+0.1s)(1+0.01s)}{(1+s)(1+0.001s)}$  (B)  $\frac{(1+0.1s)(1+s)}{(1+0.01s)(1+0.001s)}$   
 (C)  $\frac{(1+10s)(1+s)}{(1+0.1s)(1+0.001s)}$  (D)  $\frac{(1+0.001s)(1+s)}{s(1+10s)}$



**Q.2** What is the transfer function for given bode plot shown in the figure

- (A)  $\frac{4(s+0.5)^2}{(s+10)^3}$  (B)  $\frac{16000(s+0.5)^2}{(s+10)^3}$   
 (C)  $\frac{160(s+0.5)^2}{(s+10)^3}$  (D)  $\frac{1600(s+0.5)^2}{(s+10)^3}$

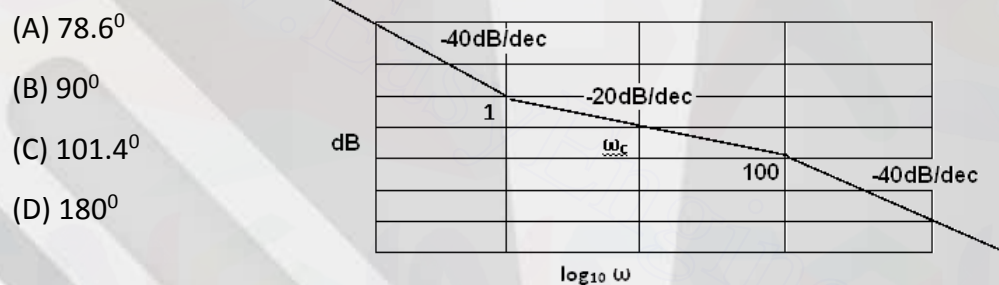


**Q.3** The asymptotic Bode plot of the transfer function  $T(s) = \frac{K}{(1 + \frac{s}{a})}$  is given in figure. The error in phase angle and dB gain at a frequency of  $\omega = 0.5a$  are respectively



- (A)  $4.9^\circ$ , 0.97 dB      (B)  $5.7^\circ$ , 3 dB      (C)  $4.9^\circ$ , 3 dB      (D)  $5.7^\circ$ , 0.97 dB

**Q.4** The asymptotic magnitude Bode plot of an open loop system  $G(s)$  with  $K > 0$  and all poles and zeroes on the  $s$ -plane is shown in the figure. It is completely symmetric about  $\omega_c$ . The minimum absolute phase angle contribution by  $G(s)$  is given by



- (A)  $78.6^\circ$       (B)  $90^\circ$       (C)  $101.4^\circ$       (D)  $180^\circ$
- Q.5** The open loop transfer function of a system is given by  $G(s)H(s) = \frac{100(s+100)}{s(s+10)}$  in the straight line approximation of the Bode plot  $|G(j\omega)H(j\omega)|$  and  $\angle G(j\omega)$  at  $\omega = 100$  rad/sec are
- (A) 0 dB and  $-\frac{3\pi}{4}$  rad      (B) 0 dB and  $\frac{\pi}{4}$  rad      (C) 20 dB and  $-\frac{3\pi}{4}$  rad      (D) 20 dB and  $\frac{\pi}{4}$  rad

## Type 8: Nyquist Stability Criterion

For Concept, refer to Control Systems K-Notes, Frequency Response Analysis

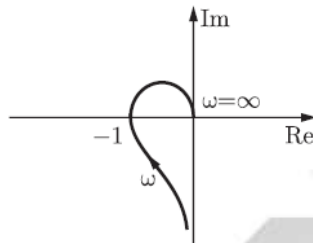
### Common Mistake:

$N = P - Z$ , we often confuse between the terms  $P$  and  $Z$  so try to remember them clearly.

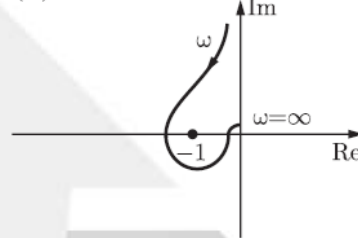
### Sample Problem 8:

Consider the following Nyquist plots of loop transfer functions over  $\omega = 0$  to  $\omega = 3$ . Which of these plots represent a stable closed loop system?

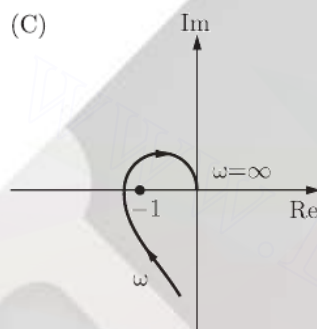
(A)



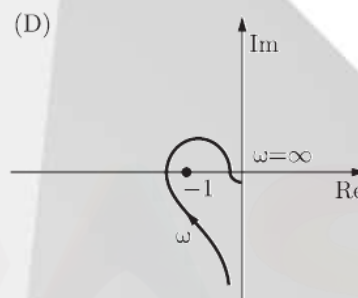
(B)



(C)



(D)



(A) (a) only

(B) all, except (a)

(C) all, except (c)

(D) (a) and (b) only

**Solution:** (A) is correct option

In the given options only in option (a) the nyquist plot does not enclose the unit circle  $(-1, j0)$ , So this is stable.

### Unsolved Problems:

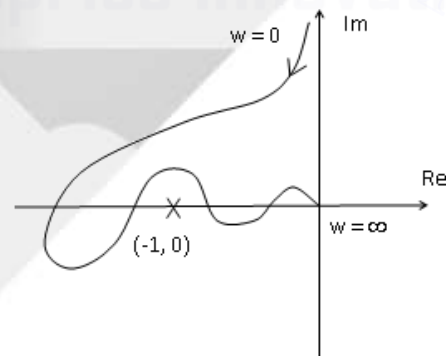
**Q.1** The nyquist plot of a closed loop system is shown in the given figure. The plot indicates that the

(A) System is unstable

(B) System is stable

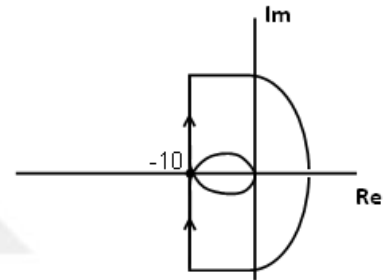
(C) System is critically stable

(D) Stability of the system can't be determined



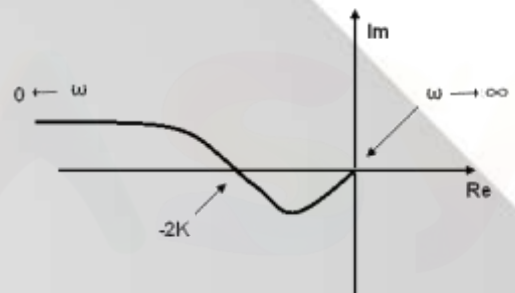
**Q.2** The Nyquist plot of  $G(s)H(s)$  which is given below. The corresponding closed loop system is unstable with two right hand poles. The number of open loop right hand poles are?

- (A) 1
- (B) 2
- (C) 3
- (D) 0



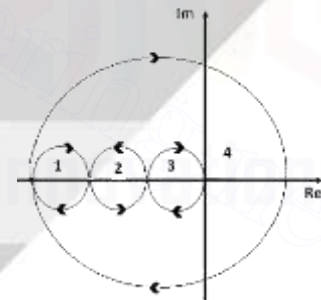
**Q.3** Suppose the Nyquist plot of the loop transfer function  $G(j\omega)H(j\omega)$  for  $\omega=0$  and  $\omega=\infty$  for a single loop feedback control system is shown in the figure. The gain  $K$  appears as a multiplying factor in  $G(s)H(s)$ . One pole of  $G(s)H(s)$  lies in the right half of the  $s$ -plane and no pole is on the  $j\omega$  axis. Which of the following statements is true in the case of closed loop stability

- (A) The closed loop system is stable for  $0.25 < K < 0.5$
- (B) The closed loop system is stable for  $K > 0.5$
- (C) The closed loop system is unstable for all values of  $K$
- (D) The closed loop system is stable for  $K = 0.25$



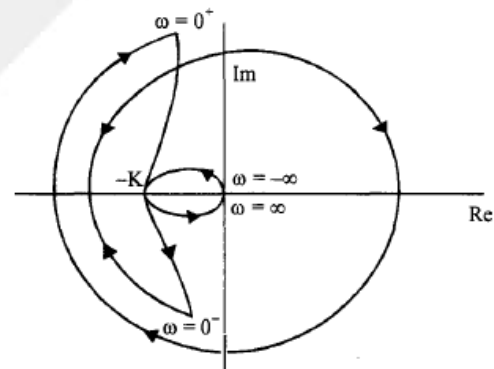
**Q.4** The Nyquist plot of  $G(s)H(s)$  which has no RHP on given below, the corresponding closed loop to be stable  $(-1, j0)$  should lie in the region

- (A) 1
- (B) 2
- (C) 3
- (D) 4



**Q.5** The Nyquist plot shown in the figure below, what is the type of the system?

- (A) 1
- (B) 2
- (C) 3
- (D) 4





## Type 9: Properties of Root-Locus

For Concept, refer to Control Systems K-Notes, Root Locus Technique

### Common Mistake:

Try to remember clearly how to distinguish between Breakaway and Break-in points.

### Sample Problem 9:

The transfer function of this system is  $G(s) = \frac{K(s^2 - 2s + 2)}{(s + 2)(s + 3)}$ . The break point is

- (A) Break-away at  $s = -1.29$
- (B) Break-in at  $s = -2.43$
- (C) Break-away at  $s = -2.43$
- (D) Break-in at  $s = -1.29$

**Solution:** (C) is correct option

The point, at which multiple roots are present, are known as break point. These are obtained from:

$$\frac{dK}{ds} = 0$$

Here, characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s^2 - 2s + 2)}{(s + 2)(s + 3)} = 0 \Rightarrow K = \frac{-(s + 2)(s + 3)}{(s^2 - 2s + 2)} = \frac{-(s^2 + 5s + 6)}{(s^2 - 2s + 2)}$$

Now, differentiating eq(1) w.r.t  $s$  and equating zero we have

$$\frac{dK}{ds} = \frac{-(s^2 - 2s + 2)(2s + 5) + (s^2 + 5s + 6)(2s - 2)}{(s^2 - 2s + 2)^2} = 0$$

$$7s^2 + 8s - 22 = 0$$

which gives  $s = +1.29$  and  $s = -2.43$  out of which  $s = -2.43$  is break-away point

### Unsolved Problems:

**Q.1** The open loop transfer function of a system is  $G(s) = \frac{K}{s(s + 4)(s^2 + 4s + 8)}$  the value of  $k$  at

( $s = -2$ ) in the root locus is

- (A) 4
- (B) 8
- (C) 16
- (D) 32

**Q.2** The OLTF of a unity feedback control system is  $G(s) = \frac{K}{(s + 1)(s^2 + 4s + 5)}$ . The angle of

departure at the pole ( $-2 - j1$ ) is

- (A)  $+60^\circ$
- (B)  $-60^\circ$
- (C)  $+45^\circ$
- (D)  $-45^\circ$

**Q.3** A unity feedback system has open loop poles at  $s = -2 \pm j2$ ,  $s = -1$ , and  $s = 0$ ; and a zero at  $s = -3$ . The angle made by the root locus asymptotes with the real and the point of Intersection of the asymptotes are

- (A)  $(60^\circ, -60^\circ, 180^\circ)$  and  $-3/2$  (B)  $(60^\circ, -60^\circ, 180^\circ)$  and  $-2/3$   
(C)  $(45^\circ, -45^\circ, 180^\circ)$  and  $-2/3$  (D)  $(45^\circ, -45^\circ, 180^\circ)$  and  $-4/3$

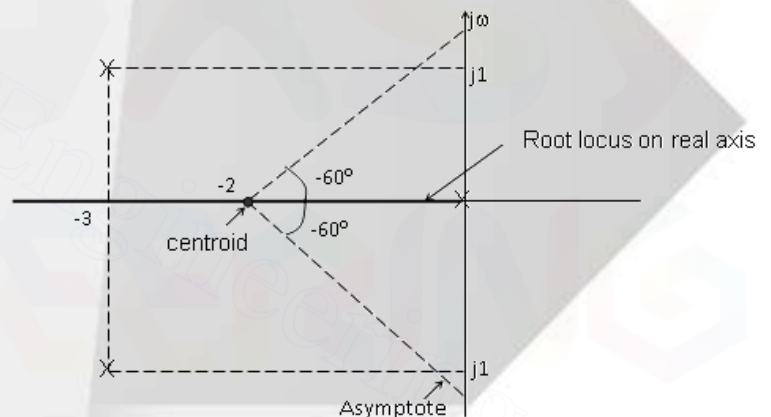
**Q.4** Consider the points  $S_1 = -3 + j4$  and  $S_2 = -3 - j2$  in the  $S$  - plane, then for a system with the open loop transfer function  $G(s)H(s) = \frac{K}{(s+1)^4}$

- (A)  $S_1$  is on the root locus, but not  $S_2$  (B)  $S_2$  is on the root locus, but not  $S_1$   
(C) Both  $S_1$  and  $S_2$  are on the root locus (D) Neither  $S_1$  nor  $S_2$  is on the root locus

**Q.5** Figure shows the asymptote, root locus on real axis and location of poles and centroid.

Break-in point of the root locus is

- (A) -3  
(B) -2  
(C) -1.18  
(D) -2.82



**Q.6** OLTF of an unity feedback system is  $\frac{K(s+1)}{s(s-1)}$ . For complex roots, the RLD is circle of centre, whose coordinates are

- (A) -0.5, 0 (B) 1, 0 (C) -1.5, 0 (D) -1, 0

## Type 10: Gain Margin and Phase Margin

For Concept, refer to Control System K-Notes, Frequency Response Analysis

### Sample Problem 10:

The frequency response of a linear system  $G(j\omega)$  is provided in the tabular form below

$ G(j\omega) $	1.3	1.2	1.0	.8	.5	.3
$\angle G(j\omega)$	$-130^\circ$	$-140^\circ$	$-150^\circ$	$-160^\circ$	$-180^\circ$	$-200^\circ$

Gain Margin and phase margin are

- (A) 6 dB and  $30^\circ$  (B) 6 dB and  $-30^\circ$   
 (C)  $-6$  dB and  $30^\circ$  (D)  $-6$  dB and  $-30^\circ$

**Solution:** (A) is correct option

Gain margin is simply equal to the gain at phase cross over frequency ( $\omega_p$ ). Phase cross over frequency is the frequency at which phase angle is equal to  $-180^\circ$ . From the table we can see that  $\angle G(j\omega_p) = -180^\circ$ , at which gain is 0.5.

$$GM = 20 \log_{10} \left( \frac{1}{|G(j\omega_p)|} \right)$$

$$= 20 \log_{10} \left( \frac{1}{.5} \right) = 6 \text{ dB}$$

Phase Margin is equal to  $180^\circ$  plus the phase angle  $\phi_g$  at the gain cross over frequency ( $\omega_g$ ). Gain cross over frequency is the frequency at which gain is unity. From the table it is clear that  $|G(j\omega_g)| = 1$ , at which phase angle is  $-150^\circ$

$$\phi_{PM} = 180^\circ + \angle G(j\omega_g)$$

$$= 180^\circ - 150^\circ = 30^\circ$$

### Unsolved Problems:

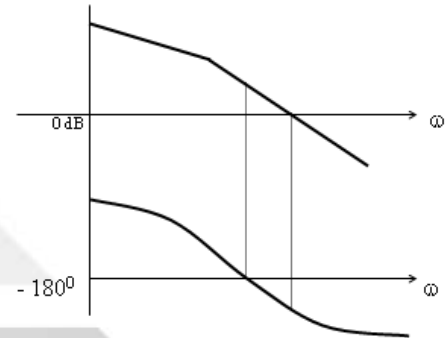
**Q.1** The open loop transfer function of a system is  $\frac{K}{s(1+0.2s)(1+0.05s)}$ . Determine the value

of 'k' such that the phase margin is  $60^\circ$ ?

- (A) 2.3 (B) 3.3 (C) 3.2 (D) 5.2

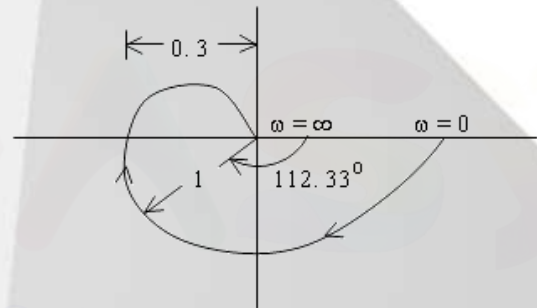
**Q.2** The Bode plot of a unity feedback system is shown. The system has

- (A) +ve P.M. and -ve G.M
- (B) +ve P.M. and +ve G.M
- (C) -ve P.M. and -ve G.M
- (D) +ve P.M. and +ve G.M



**Q.3** A unity feedback system has OLTF  $G(s)$ . Polar plot is shown in the figure. The gain margin and phase margin are

- (A)  $GM = -0.3$ ,  $PM = 112.33^\circ$
- (B)  $GM = 0.3$ ,  $PM = 112.33^\circ$
- (C)  $GM = 3.33$ ,  $PM = 67.67^\circ$
- (D) None of the above



**Q.4** The open loop transfer function of a unity feedback control system is given as  $G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$ . The phase crossover frequency and the gain margin are, respectively.

- (A)  $\frac{1}{\sqrt{T_1 T_2}}$  and  $\frac{T_1 + T_2}{T_1 T_2}$
- (B)  $\frac{1}{T_1 + T_2}$  and  $\frac{T_1 + T_2}{T_1 T_2}$
- (C)  $\frac{1}{\sqrt{T_1 T_2}}$  and  $\frac{T_1 T_2}{T_1 + T_2}$
- (D)  $\frac{1}{T_1 + T_2}$  and  $\frac{T_1 T_2}{T_1 + T_2}$

**Q.5** In the  $G(s)H(s)$ -plane, the Nyquist plot of the loop transfer function  $G(s)H(s) = \frac{\pi e^{-0.25s}}{s}$  passes through the negative real axis at the point

- (A)  $(-0.25, j0)$
- (B)  $(-0.5, j0)$
- (C) 0
- (D) 0.5

## Type 11: Compensators

For Concept, refer to Control System K-Notes, Design of Control Systems

**General Tip:** Memorize the formulas for maximum phase shift and frequency corresponding to maximum phase shift.

### Sample Problem:

The transfer functions of two compensators are given below:

$$C_1 = \frac{10(s+1)}{(s+10)}, \quad C_2 = \frac{(s+10)}{10(s+1)}$$

Which one of the following statements is correct?

- (A)  $C_1$  is lead compensator and  $C_2$  is a lag compensator
- (B)  $C_1$  is a lag compensator and  $C_2$  is a lead compensator
- (C) Both  $C_1$  and  $C_2$  are lead compensator
- (D) Both  $C_1$  and  $C_2$  are lag compensator

**Solution:** (A) is correct option

For  $C_1$  Phase is given by

$$\theta_{C_1} = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$\theta_{C_1} = \tan^{-1} \left[ \frac{\omega - \frac{\omega}{10}}{1 + \frac{\omega^2}{10}} \right] = \tan^{-1} \left[ \frac{9\omega}{10 + \omega^2} \right] > 0 \text{ (Phase lead)}$$

Similarly for  $C_2$ , phase is

$$\theta_{C_2} = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}(\omega)$$

$$\theta_{C_2} = \tan^{-1} \left[ \frac{\frac{\omega}{10} - \omega}{1 + \frac{\omega^2}{10}} \right] = \tan^{-1} \left[ \frac{-9\omega}{10 + \omega^2} \right] < 0 \text{ (Phase lag)}$$

### Unsolved Problem:

**Q.1** Which of the following is the transfer function of a lead compensation network.

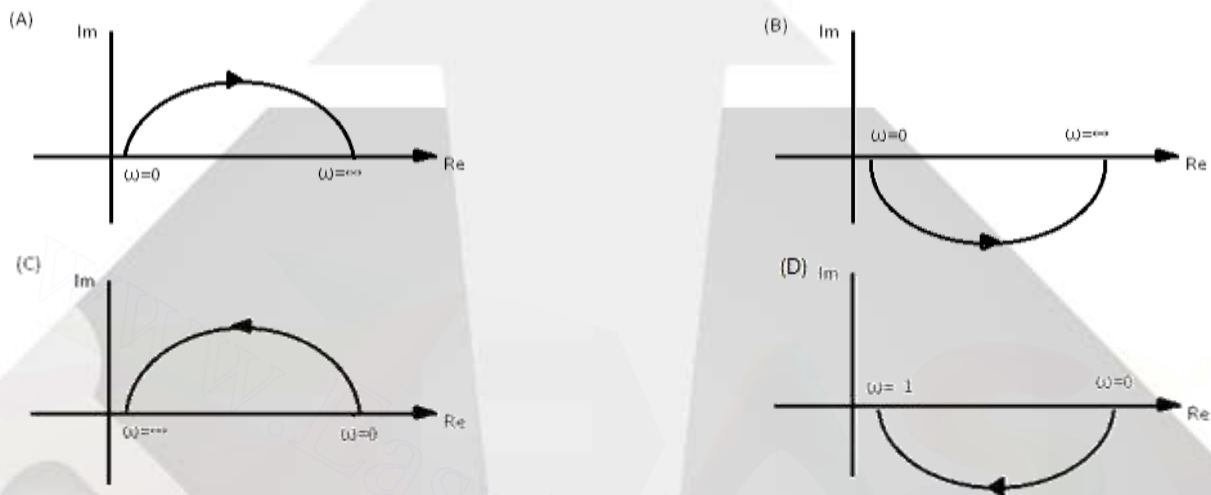
- (A)  $\frac{(s+5)}{(s+8)}$
- (B)  $\frac{(s+8)}{(s+5)}$
- (C)  $\frac{s(s+5)}{(s+8)}$
- (D)  $\frac{(s+8)}{s(s+5)}$



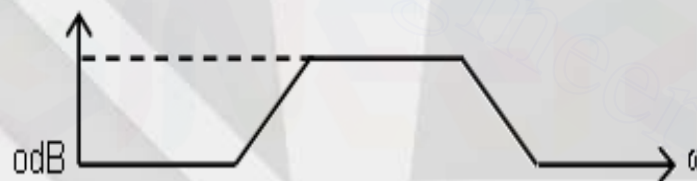
**Q.2** Maximum phase lead of the compensator  $D(s) = \frac{(0.5s + 1)}{(0.05s + 1)}$  is

- (A) 52 at 4 rad/sec (B) 52 at 10 rad/sec (C) 55 at 12 rad/sec (D) None of the above

**Q.3** Which one of the polar diagram corresponds to a lag network?



**Q.4** The frequency response of controller is given below. The controller is



- (A) Lead (B) Lag (C) Lead-lag (D) Lag-lead

**Q.5** The controller of the form  $G_c(s) = \frac{K(s + a)}{(s + b)}$ . A first order compensator is designed for a plant

with the transfer function  $G(s) = \frac{1}{s(s + 3)}$  such that the un-damped natural frequency and the

damping ratio of the closed loop second order system are 2 rad/sec and 0.5 respectively. When the steady state error to a unit step input is zero then the controller parameters are

- (A)  $K=4, a=3, b=2$  (B)  $K=1, a=3, b=2$  (C)  $K=4, a=2, b=3$  (D)  $K=1, a=2, b=3$

## Type 12: State Variable Analysis

For Concept, refer to Control Systems K-Notes, State Variable Analysis

### Sample Problem 12:

A system is described by the following state and output equations

$$\frac{dx_1(t)}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t), \quad y(t) = x_1(t)$$

when  $u(t)$  is the input and  $y(t)$  is the output. The system transfer function and the state-transition matrix of the above system is

$$(A) \frac{(s+2)}{(s^2+5s-6)}, \begin{bmatrix} e^{-3t} & 0 \\ e^{-3t} + e^{-2t} & e^{-2t} \end{bmatrix}$$

$$(B) \frac{(s+3)}{(s^2+5s+6)}, \begin{bmatrix} e^{-3t} & e^{-2t} + e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

$$(C) \frac{(2s+5)}{(s^2+5s+6)}, \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

$$(D) \frac{(2s-5)}{(s^2+5s-6)}, \begin{bmatrix} e^{3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

**Solution:** (C) is correct option

$$\text{T.F.} = C[sI - A]^{-1}B + D$$

where

$$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0]$$

$$[sI - A] = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+3)(s+2)}$$

$$\text{T.F.} = C[sI - A]^{-1}B = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}}{(s+3)(s+2)}$$

$$\text{T.F.} = \frac{(2s+5)}{(s^2+5s+6)}$$

State-transition matrix

$$e^{At} = L^{-1} \left\{ [sI - A]^{-1} \right\}$$

$$e^{At} = L^{-1} \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+3)(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix},$$

$$e^{At} = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

**Unsolved Problems:**

**Q.1** Given the system

$$\dot{X} = \begin{bmatrix} 0 & 0 & -20 \\ 1 & 0 & -24 \\ 0 & 1 & -9 \end{bmatrix} X + \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} X$$

The characteristic equation of the system is

- (A)  $s^3 + 20s^2 + 24s + 9 = 0$  (B)  $s^3 + 9s^2 + 24s + 20 = 0$   
 (C)  $s^3 + 24s^2 + 9s + 20 = 0$  (D) None of these

**Q.2** For a system represented by state equation  $\dot{x}(t) = Ax(t)$ , response is  $x(t) = \begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$

When  $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$  and  $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$  When  $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ , system matrix 'A' is

- (A)  $\begin{bmatrix} 1 & 0 \\ -3 & -2 \end{bmatrix}$  (B)  $\begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}$  (C)  $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$  (D)  $\begin{bmatrix} 1 & 0 \\ -5 & -3 \end{bmatrix}$

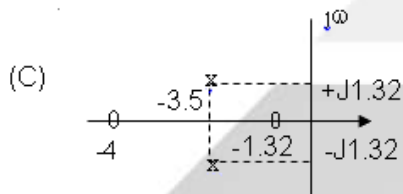
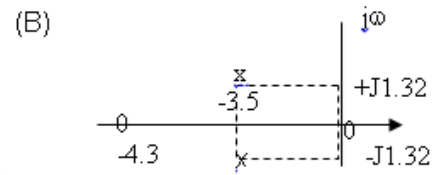
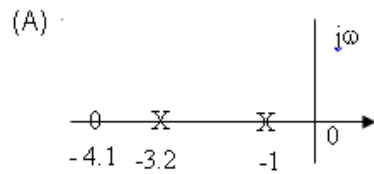
**Q.3** The system is represented by state equations as follows.

$$\dot{X}_1 = -4X_1 - X_2 + 3u$$

$$\dot{X}_2 = 2X_1 - 3X_2 + 5u$$

$$Y = X_1 + 2X_2$$

The poles and zeros locations are



(D) None

**Q.4** The dynamic model of a pendulum is given by  $\frac{d^2\theta}{dt^2} + 4\theta = T$ , where  $\theta$  is the displacement in rad/sec and  $T$  is the applied torque in N-m. Its representation in the time scale state variable form  $\dot{X} = \alpha X + \beta u$  can have the constants?

(A)  $\alpha = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$ ,  $\beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(B)  $\alpha = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$ ,  $\beta = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(C)  $\alpha = \begin{bmatrix} 0 & 0 \\ 4 & 1 \end{bmatrix}$ ,  $\beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(D)  $\alpha = \begin{bmatrix} 0 & 0 \\ -4 & 1 \end{bmatrix}$ ,  $\beta = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

**Q.5** Obtain the response  $y(t)$  of the following system

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u, \quad \begin{bmatrix} X_1(0) \\ X_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y = [1 \ 0] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

(A)  $e^{-0.5t} \cos(0.5t)$

(B)  $e^{-2t} \cos(0.5t)$

(C)  $e^{-0.5t} \sin(0.5t)$

(D)  $e^{-2t} \sin(0.5t)$

## Type 13: Controllability and Observability

For Concept, refer to Control Systems K-Notes, State Variable Analysis

### Sample Problem 13:

The second order dynamic system  $\frac{dx}{dt} = PX + Qu$ ,  $Y = RX$  has the matrices P, Q and R as follows:

$$P = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}, Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, R = [0 \quad 1]$$

The system has the following controllability and observability properties:

- (A) Controllable and observable
- (B) Not controllable but observable
- (C) Controllable but not observable
- (D) Not controllable and not observable

**Solution:** (C) is correct option

Controllability Matrix:

$$C = [Q \quad PQ]$$

$$Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; PQ = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$\det(C) = 3 \times 0 - 1 = -1 \neq 0$  hence controllable

Observability Matrix

$$O = \begin{bmatrix} R \\ RP \end{bmatrix}$$

$$R = [0 \quad 1] : RP = [0 \quad 1] \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} = Q [0 \quad -3]$$

$$O = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$$

$\det(O) = 0$  not observable



## Unsolved Problems:

**Q.1** A state variable model of a system is given by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y = [1 \ 0] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

The system is

- (A) Controllable and observable (B) Controllable but unobservable  
(C) Observable and uncontrollable (D) Uncontrollable and unobservable

**Q.2** Consider the system  $\frac{dx}{dt} = AX + Bu$  with  $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} p \\ q \end{bmatrix}$  where  $p$  and  $q$  are arbitrary real numbers. Which of the following statements about the controllability of the system is true?

- (A) The system is completely state controllable for any nonzero values of  $p$  and  $q$ .  
(B) Only  $p=0$  and  $q=0$  result in controllability.  
(C) The system is uncontrollable for all values of  $p$  and  $q$ .  
(D) We cannot conclude about controllability from the given data.

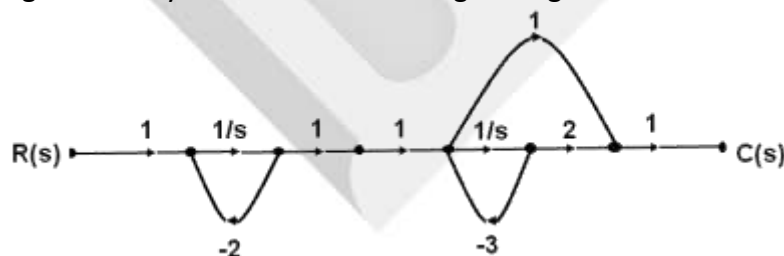
**Q.3** The state variable description of an LTI system is given by

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, \quad y = [1 \ 0 \ 0] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

where  $y$  is the output and  $u$  is the input. The system is controllable for

- (A)  $a_1 \neq 0, a_2 = 0, a_3 \neq 0$  (B)  $a_1 = 0, a_2 \neq 0, a_3 \neq 0$   
(C)  $a_1 = 0, a_2 \neq 0, a_3 = 0$  (D)  $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

**Q.4** The state diagram of a system is shown in the given figure:



The system is

- (A) Controllable and observable (B) Controllable but not observable  
(C) Not Controllable but observable (D) Neither Controllable nor observable

**Q.5** Consider the system defined by 
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, \quad y = [C_1 \ C_2 \ C_3] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

except for an obvious solution choice  $C_1=C_2=C_3=0$ , find an example of a set of  $C_1, C_2, C_3$  that will make the system unobservable

- (A)  $C=[1 \ 1 \ 1]$  (B)  $C=[0 \ 1 \ 1]$  (C)  $C=[0 \ 1 \ 0]$  (D)  $C=[1 \ 1 \ 0]$

## Type 14: Steady State Error

For Concept, refer to Control Systems K-Notes, Time Response Analysis

### Common Mistake:

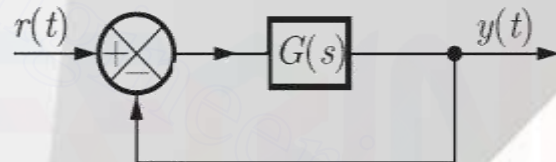
We do not need to convert a non-unity feedback system to a feedback one to find the steady state error using error constants.

### Sample Problem 14:

A unity feedback is provided to the system  $G(s) = \frac{1}{s(s+2)}$  to make it a closed loop system as

shown in figure .For a unit step input  $r(t)$ , the steady state error in the input will be

- (A) 0  
(B) 1  
(C) 2  
(D) 3



**Solution:** (A) is correct option

$$G(s) = \frac{1}{s(s+2)}, \quad H(s) = 1$$

$$\text{Error to step input} = \frac{1}{1 + K_p}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = \infty$$

$$e_{ss} = \frac{1}{1 + \infty} = 0$$

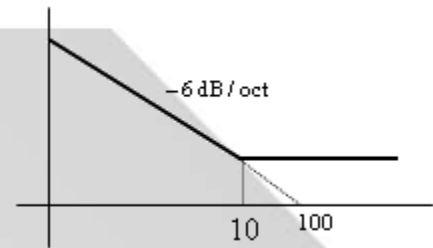
$$e_{ss} = 0$$

## Unsolved Problems:

**Q.1** The open loop transfer function of a system is  $\frac{K}{s^2(s+1)(s+10)}$ . Determine the value of 'k' such that steady state error due to unit acceleration input is 0.2  
 (A) 10 (B) 0.1 (C) 100 (D) 0.2

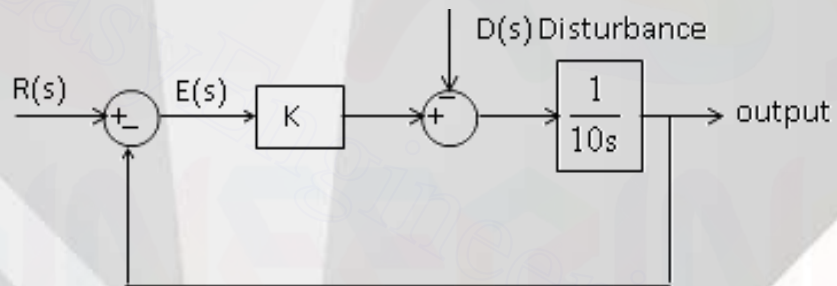
**Q.2** The Bode plot of a OLTF is given below  
 The steady state error for unit ramp input is

- (A) 10  
 (B) 0.1  
 (C) 0.01  
 (D) 0.001



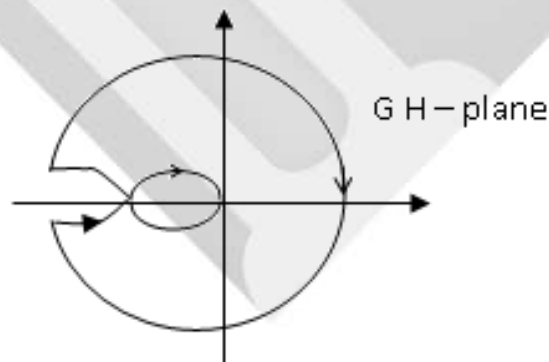
**Q.3** Steady state error to both input  $R(s) = \frac{1}{s}$  and disturbance  $D(s) = \frac{1}{s}$  acting simultaneously is?

- (A)  $\frac{11}{K}$   
 (B)  $\frac{1}{K}$   
 (C)  $\frac{10}{K}$   
 (D) 0



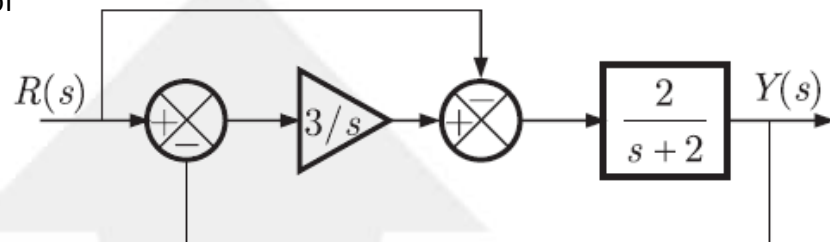
**Q.4** Nyquist plot a certain stable system is given below. The acceleration error coefficient is

- (A) 0  
 (B)  $\infty$   
 (C)  $-\infty$   
 (D)  $10^2$



**Q.5** When subject to a unit step input, the closed loop control system shown in the figure will have a steady state error of

- (A) -1.0
- (B) -0.5
- (C) 0
- (D) 0.5



## Answer key

	1	2	3	4	5	6
Type 1	D	B	B	D	A	B
Type 2	A	C	B	A	C	A
Type 3	A	A	A	D	C	
Type 4	A	A	D	B	D	C
Type 5	C	B	B	C	A	A
Type 6	B	A	C	C	C	
Type 7	A	B	A	D	A	
Type 8	B	D	B	B	C	
Type 9	C	C	B	B	C	D
Type 10	A	A	C	A	B	
Type 11	A	D	D	C	A	
Type 12	B	C	B	A	C	
Type 13	A	C	D	B	D	
Type 14	A	C	B	D	C	



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